# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION - **MATHEMATICS**

SECOND SEMESTER – **NOVEMBER 2013**

**MT 2812 - PARTIAL DIFFERENTIAL EQUATIONS**

Date : 18/11/2013 Dept. No. Max. : 100 Marks
Time : 1:00 - 4:00

**Answer ALL the Questions:**

1. (a) Using Jacobi's method solve $z = p^2x + q^2y$. (5)

OR

(b) Solve $p^2q(x^2 + y^2) = p^2 + q$. (5)

(c) Show that the equations $f(x, y, z, p, q) = 0$ and $g(x, y, z, p, q) = 0$ are compatible if $\frac{\partial(f,g)}{\partial(x,p)} + \frac{\partial(f,g)}{\partial(y,q)} = 0$. Verify that the equations $p = P(x, y)$, $q = Q(x, y)$ are compatible if $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$. (15)

OR

(d) Find the characteristics of the equation $pq = z$, and determine the integral surface which passes through the parabola $x = 0$, $y^2 = z$. (15)

2. (a) If $f$ and $g$ are arbitrary functions show that $u = f(x - vt + i\alpha y) + g(x - vt - i\alpha y)$ is a solution of $u_{xx} + u_{yy} = \frac{1}{c^2}u_{tt}$ provided $\alpha^2 = 1 - \frac{v^2}{c^2}$. (5)

OR

(b) Solve $(D^2 - 2DD' + D'^2)z = x^3$. (5)

(c) Verify that the Green's function for the equation $\frac{\partial^2 z}{\partial x \partial y} + \frac{2}{x+y}\left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\right) = 0$ subject to $z = 0$, $\frac{\partial z}{\partial x} = 3x^2$ on $y = x$ is given by $w(x, y, \xi, \eta) = \frac{(x+y)(2xy+(\xi-\eta)(x-y)+2\xi\eta)}{(\xi+\eta)^3}$. (15)

OR

(d) Reduce the equation $(n-1)^2\frac{\partial^2 z}{\partial x^2} - y^{2n}\frac{\partial^2 z}{\partial y^2} = ny^{2n-1}\frac{\partial z}{\partial y}$ to canonical form and solve it (15)

3. (a) Obtain the Poisson's equation. (5)

OR

(b) Derive one-dimensional wave equation. (5)

(c) State and prove Interior Dirichlet Problem for a Circle. (15)

OR

(d) Obtain the solution of Diffusion equation in spherical coordinates. (15)

4. (a) Solve the wave equation given by $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, -\infty < x < \infty$, subject to the initial conditions $u(x,0) = f(x), -\infty < x < \infty, \frac{\partial u}{\partial t}(x,0) = 0$. (5)

OR

(b) Show that the Green's function $G(\bar{r}, \bar{r'})$ has the symmetry property. (5)

(c) Obtain the solution of the interior Dirichlet problem for a sphere using the Green's function. (15)

OR

(d) State and prove Helmholtz Theorem. (15)

5. (a) Find the iterated Kernel for the Kernel $K(x, t) = e^x \cos x$; $a = 0$, $b = \pi$. (5)

OR

(b) Using Fredholm determinants, find the resolvent kernel when $K(x, t) = e^{t+x}$, $a = 0$, $b = 1$. (5)

(c) Solve the symmetric integral equation $y(x) = (x+1)^2 + \int_{-1}^{1} (xt + x^2t^2) y(t) dt$. (15)

OR

(d) State and prove Hilbert- Schmidt theorem. (15)